Plain Vanilla SPX-Options Hedging:  
The Effect of Smile-Adjustments and the Lark versus Owl Question.  
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The early bird gets the worm.  
(Benjamin Franklin)

Abstract:  
Compared with the veritable explosion of research on pricing OTC options with path-dependent and/or exotic pay-offs there has been relatively little research on hedging vanilla options. Yet exchange trading on standard vanilla options is much more active than trading on OTC products. ([1]). This working-paper tests for SPX options the effect of vega adjusted delta-hedging on several trading strategies developed in a previous working paper [2]. During volatile markets risk reduction from regime-dependent delta hedging is much greater than during tranquil periods ([1],[3],[4],[5]). It is shown that a very simple regime classification scheme developed in ([2],[6]) (slightly) improves the hedging-performance.  
In the literature hedging is usually performed daily. The simple question “at what time of the day” is up to my knowledge never addressed. The second part of this paper deals with the Lark versus Owl question. Is it better to do the daily hedge after the open or before the close? The available high-frequency time-series are too short and sparse to answer this question definitely. But there is some evidence that the Lark has an edge.

Revision 1:  
When working on a GARCH Model for addressing the question mentioned in “Further Work” I noticed a bug in the Lark v. Owl calculation. In Revision 1 the complete Lark v. Owl Part is replaced by the new results. The general message is now: The available high-frequency time-series are too short and sparse to answer this question definitely. According the limited data the Owl has on a few turbulent days an edge, otherwise the performance is very similar.

Introduction:  
The Black-Scholes options model assumes a constant volatility. It is well known since 1987 that index-options are highly-negatively correlated with the market-movement. The correlation of the SPX and the VIX is between -0.8 and -0.9. The relation is in fact non-linear. The VIX explodes in times of troubles.

If volatility is time-varying and correlated with the underlying stock returns delta must not only control for the direct impact of the underlying price change on the option price, but also for the indirect impact of the volatility change which is correlated with the underlying price change. Assume volatility to be a deterministic function of $S$, $K$, and $T$. Then, by the chain rule, the delta of the option is given by

$$
\Delta = \frac{\delta C}{\delta S} + \frac{\delta C}{\delta \sigma} * \frac{\delta \sigma}{\delta S} \quad (1)
$$

$C$ … Options-Price  
$S$ … Base-Price.
With the Greeks of the Black-Scholes model equation (1) can be written as:

$$\Delta = \Delta_{BS} + \text{vega}_{BS} \times \frac{\delta \sigma}{\delta S} \quad (2)$$

Since vega is for plain-vanilla options always positive, equation (2) shows that in the case of a negative correlation between stock returns and volatility changes, the adjusted delta should be smaller. The unknown dependence of volatility on the underlying stock price $\delta \sigma/\delta S$ can be approximated by the slope of the current volatility smile $\partial \sigma/\partial K$ ([4]). In fact this approximation holds only, if one assumes the sticky-implied-tree rule ([7]). It is argued in [3] and [7] that this is only the case in a volatile market-regime. Due to it's simplicity equation (2) is the rule of choice in [1],[4],[5].

If one has a reliable regime classifier at hand, one can and should switch on and off the vega adjustment.

**Methods and Data:**

In [2] I developed several simple options trading strategies. It is a non-trivial task to get long time-series of options data.

*Note: Actually it is quite simple to get them, it is just a matter of enough money.*

For the study in [2] 5 years of daily SPX options data were bought. The data were cleaned and I developed data-structures to access this large amount of data effectively. The data range from 2007-12-11 to 2012-12-11. The first part of this working paper reuses this data collection. The second part uses recent high-frequency data which were collected online from IQFeed in the last months.

In the academic literature one holds one call and hedges with the fractional part of the underlying index. In real-trading life one has to hedge SPX options with eMini futures. Obviously one can't trade fractional futures. This study takes the correct $ multipliers of the options (100) and the eMinis (50) into account. The delta-sum is rounded to the nearest integer. One has a quantization effect. To test this effect, trading was done with a volume of 10, 50 and 100 options per trade. The results differ only slightly. The overall trend is: The performance of the smaller volume is slightly better than the larger one. Quantization improves performance. If one trades only 10 options, a small change in delta usually has no effect on the eMini position. This result is inline with theoretical models. In [8] it is shown that there is a hysteresis like curve around the delta-function. If delta stays inside the hysteresis bounds one keeps the hedge-ratio constant. Quantization has the same effect.

**The Implied-Volatility-Term-Structure (IVTS):**

In [2] the concept of the implied-volatility-term-structure is used to classify market regimes. Several variants of the concept are defined and analyzed in [5]. For the current work I will stick to the original definition in [2]. The IVTS is the ratio between the 1-month and 3-month implied volatility index.

$$\text{IVTS}(t) = \frac{\text{VIX}(t)}{\text{VXV}(t)} \quad (3)$$

The IVTS is certainly more useful than using the VIX (or VXV) alone. One can consider the division by the slower moving VXV as a sort of normalization. The VXV measures the general level of volatility, the VIX reacts to the current situation.

**OTM-Put Writing:**

In [2] one of the most interesting strategies was OTM-Put writing. It is well known that OTM-Puts have a high risk premium. Starting at 2010-01-03 one writes OTM Puts with a maturity of 31 trading days. Trading is done till 2012-12-11 (the end of the available time-series). OTM is measured by the
options delta. OTM-Puts with a strike 0 modulo 25 are more actively traded (if the S&P is at 1800, the 1725P has a much higher volume than 1730P or 1720P). Therefore only strikes with 0 mod 25 are considered. One writes the lowest strike with delta < -0.15.

One starts with an index value of 500,000$. This is an arbitrary number used in previous studies. The volume is $4 \times 10^{-5}$ times the current index value. For the initial value of 500,000$ one writes 20 Puts. If the index grows to 600,000, one would write 24 Puts.

Note: In [2] a constant volume was used. The new rule is more inline with trading-practice.

The options are kept till expiry. One has either a position of -20 or -40 options in ones portfolio.

Several possibilities to improve these rules were analyzed in [2]. The current task is to investigate the effectiveness of hedging-approaches. So we stick to the most basic setting.

There are within the considered time-range 3 major events for an OTM Put writing strategy. The Japanese earthquake in March 2010. The flash-crash in May 2010 and the severe market-crash in August 2011.

The academic literature tries to minimize the hedging-error. This is for a fund not the relevant measure.

One wants a high return combined with a modest relative maximum drawdown. The performance of the hedging-strategies is therefore measured in these more interesting units.

Graphic-1: OTM-Put: BS-Hedge (orange) and Smile-Adjusted (yellow) 2010-01-03 to 2012-12-11

Graphic-1 compares the performance of the plain BS-hedge (orange) with the smile-adjusted hedge of equation (2). The plain hedge has an overall performance of 45.8%. The max. relative drawdown (blue chart in Graphic-1) is 27.6% on 2011-08-18. The smile-adjusted hedge performs clearly better. The overall performance is 49.3% with a max. relative drawdown of 19.8%. But the improved hedge during the crashes is not for free. One shorts in the smile-adjusted version more futures. This costs some extra
money in upwards trending markets. The extra hedge is in a bull market a waste of money.

Graphic-2 shows a market-regime adjusted hedge-strategy. If the IVTS is below 0.91 (a bull market) the plain BS-hedge is used. Between 0.91 and 1.00 (a sideways market) the hedge is smile adjusted. If the IVTS is above 1.0 (a crash) the smile adjustment is multiplied by 1.4. This should compensate for the non-linear increase of the implied volatility in times of troubles. The adaptive version performs somewhat better than the simple smile adjustment. The overall win is 50.7%, the max. relative drawdown is reduced to 18.0%. The adjustment is not entirely for free. In the aftershocks of the summer 2011 crash the IVTS is still relative high. The increased hedge ratio drags the profit down. But the behavior is in bull markets identical to plain BS-hedging.

Graphic-3 and 4 show an aggressive version of this idea. If the IVTS is below 0.91, one does not hedge at all. This increases the overall profit to 67.3%. The max. drawdown is in August 2011 the same as before. But there is an additional risk for a bolt out of the blue. Such a bolt occurred on 2012-07-14 (red line in Graphic-3 and 4). The blue charts in Graphic-3 and 4 show the different drawdowns of the hedged and the naked position. The hedged position has only a minor hump, the naked Puts loose more than 7%.

Note: The flash-crash was not a bolt out of the blue. The IVTS was already before in the red zone.
Graphic-3: OTM-Put: BS-Hedge (orange) and Smile-Aggressive (yellow) 2010-01-03 to 2012-12-11

Graphic-4: OTM-Put: BS-Hedge (orange) and Smile-Aggressive (yellow) 2010-01-03 to 2012-12-11
Hedging the Kir-Put:
The Kir strategy closes the position once the IVTS is above 1.0. One stays in turbulent market conditions on the sideline. This is a trivial but quite effective hedge.

Consequently the difference between the plain BS-hedge and the smile adjusted almost disappears. The adjusted hedge does somewhat better in the aftershocks of the flash-crash (red line in Graphic 5). On the other side there is the extra hedge drag of the adjustment in bull markets. The plain hedge has an overall win of 47.1% and a max. relative drawdown of 6.8% at 2010-07-26. The values for the smile-adjusted hedge are a win of 45.9% and a drawdown of 5.2%. The adaptive adjustment does again somewhat better. The overall win is 48.8% and the drawdown is further reduced to 4.1%.

Graphic-6 shows the behavior of the aggressive hedge strategy. The overall win increases to 65.5%. The Kir does of course not avoid the bolt out of the blue problem on 2012-07-14.

One could think about to trade the position completely naked. But this does not improve the performance. The naked version looses considerable in the aftershocks of the flash-crash (Graphic-7).
Graphic-6: OTM-Put-Kir: BS-Hedge (orange) and Aggressive (yellow) 2010-01-03 to 2012-12-11

Graphic-7: OTM-Put-Kir: Naked (orange) and Aggressive (yellow) 2010-01-03 to 2012-12-11
OTM-Strangle Writing:
For this strategy a Call with the same characteristics is added to the Put. Calls have usually no risk-premium. But they can add some extra money to the Put position and are most of the time easier to hedge than Puts. This was the best strategy in [2]. This is also confirmed in Graphic-8. The plain hedge has an overall performance of 59.5% (OTM-Put 45.8%). The max. relative drawdown (blue chart in Graphic-1) is 28.9% (27.6%) on 2011-08-18. The smile-adjusted hedge performs clearly better. The overall performance is 65.2% (49.3%) with a max. relative drawdown of 21.5% (19.8%). The effect of the smile-adjustment is larger than for the Put case.

Graphic-8: OTM-Strangle: BS-Hedge (orange), Smile-Adjusted (yellow) 2010-01-03 to 2012-12-11

Graphic-9 shows the performance of the smile-adaptive strategy. The performance increases to 71.86% with a max. relative drawdown of 21.9%. The maximum drawdown is now not in the summer 2011 crash but shifts to the flash-crash shock-waves. The strategy suffers from the sharp up- and downs of that period.

The aggressive smile-adjustment improves the performance to 105.6% with a max. relative drawdown of 18.9%. One has – of course – the hump at 2012-07-14 which is not present in the other strategies.
Graphic-9: OTM-Strangle: BS-Hedge (orange) and Smile-Adaptive (yellow) 2010-01-03 to 2012-12-11

Graphic-10: OTM-Strangle: BS-Hedge (orange) and Smile-Aggressive (yellow) 2010-01-03 to 2012-12-11
**Hedging the Kir-Strangle:**
For the Kir-Strangle one closes like before for the position if the IVTS is above 1.0. The overall pattern is identical to the Kir-Put. The simple smile-adjustment does not improve the result. The adaptive strategy has a slight edge. The aggressive version is – at the cost of some extra risk – clearly better. The results are hence not repeated in full detail.
Lark or Owl?
The historic simulation hedges at the close. There is not other choice, because there are only daily-close data available. The same was done in real-life trading of the Sibyl-fund. One does of course not hedge at the close, but shortly before. Sibyl-trader Siddharth Bhatia noted that according his feeling it would be better to hedge shortly after the open. This investigation addresses this question. Unfortunately I have only a limited amount of high-frequency option data available. The result is therefore only a hint. Hedging after the open is called the Lark, before the close the Owl.

Note: This part is completely rewritten in Revision 1. When working on a new GARCH model I noticed, that there was a bug in the calculation. The hedge calculation used not the Options value at the time of hedging, but in all cases the daily close value. This favors the Lark. The Lark knows in the morning already the options Close value. The SPX value was correct.

Graphic-11 shows the results for the Kir OTM-Strangle strategy which was actually traded from 2013-05-31 to 2014-02-25. Only the options trades are real-life input, the hedging positions are calculated in the historic simulations (the real-life hedges are a mixed bag). Hedging is done with plain BS-Delta. The result does not confirm Sid's observation. Especially at 2013-08-14 (red line in Graphic-11) the Lark is considerable worse. The S&P was already dropping over the day and fall down sharply overnight. The jump was for the Owl less than for the Lark. But otherwise there is not much difference (Graphic-12).

Note: The original conclusion was: But otherwise the Lark was doing a much better job. The Lark had due to the bug an advantage of knowing the Options close value.

None of the adjustments perform better than the plain-vanilla BS-Hedging. One of the reasons is that the smile calculations are for the first historic-simulation more reliable. I had daily-data for all traded strikes. This is not the case for the second series with HF-data.

Note: GARCH-Hedging has a slight edge. But this is the topic of a forthcoming working-paper.
Graphic-11: Actual Trades: Lark (orange) and Owl (yellow) BS-Hedging 2013-05-31 to 2014-02-25.

Graphic-12: Actual Trades: Lark (orange) and Owl (yellow) BS-Hedging 2013-08-16 to 2014-02-25.
To test this question further I hand picked according simple rules another series of options-positions. One selects at every 3rd Wednesday at 10:00 a Strangle. The strikes are 100 or 125 points apart. If the S&P is at 1760, one would pick the 1800 Call and 1700 Put. One sells the position back at 10:00 on the last trading day (the Thursday before the 3rd Friday of the following month). There is additionally the Kir criterion. If the IVTS is at a given day at 10:00 above 1.0, the position is closed premature. Graphic-13 shows the result. The Owl is at the beginning, in the somewhat turbulent times of June 2013, better. But since Jul. 2013 the hedging performance is almost identical (Graphic 14). The adjustments are (besides GARCH) no improvement.

Note: The original conclusion was: The Lark has still an edge, but the difference is much smaller than for the real-traded position.

Graphic-13: 3rd Wednesday Trades: Lark (orange) and Owl (yellow) BS-Hedging 2013-05-31 to 2014-02-25.
In Graphic-15 the position is already entered on the 1\textsuperscript{st} Wednesday. The maturity is now about 6 weeks. On one occasion the position was entered on the 2\textsuperscript{nd} Wednesday, because on the 1\textsuperscript{st} the IVTS was above 1.0. The Owl has in June 2013 a clear edge. But if one considers the performance from July 2013 on, the Lark is slightly ahead (Graphic-16).
Graphic-15: 1\textsuperscript{st} Wednesday Trades: Lark (orange) and Owl (yellow) BS-Hedging 2013-05-31 to 2014-02-25

Graphic-16: 1\textsuperscript{st} Wednesday Trades: Lark (orange) and Owl (yellow) BS-Hedging 2013-07-01 to 2014-02-25
In Graphic-17 only the 1st Wednesday Puts are traded. It is an OTM Put strategy. Generally the OTM-Puts are much easier to trade in the considered time range. It was most of the time a strong bull market with no major crash in between. For this portfolio the Lark performs somewhat better.

For the Put-Position the aggressive (and GARCH) hedge has a clear edge over the BS-hedging (Graphic-18). The Puts are during the strong bull-rally naked. There are downward spikes like on 2013-11-07 (red line in Graphic-18) in between. But the overall performance is clearly superior. This holds also for the other portfolios if one trades the Puts only. The Aggressive hedge does not work for the Strangles, because the naked Calls get during the rallies into the money.
Graphic-18: 1\textsuperscript{st} Wednesday Put: Lark BS-Hedge (orange) and Aggressive (yellow) 2013-05-31 to 2014-02-25

**Conclusions:**
Adjusting the BS-Delta hedge has its greatest positive effect during crashes. But the Kir strategy is during a crash anyway out of the market. The adjustment nevertheless improved the performance in the first time-series. It did not work in the second one. This can be due to a lack of available strikes to calculated the smile. But it should be also noted that the market conditions were in the second period considerable different. There is probably no single strategy which works all the time best.

The aggressive hedge, which changes to a naked position once the IVTS falls below 0.91, seems to be an interesting alternative in a Put-only scenario. There is the risk of a bolt out of the blue. If this happens intraday one can close the position or hedge accordingly. This was not done in the historic simulation. The S&P-500 can of course also nosedive overnight. But in all major crashes of the recent past the IVTS is long before over 0.91. A possible scenario is a severe natural catastrophe.

The Lark versus Owl question seems to be within the framework of the Kir strategy more interesting. But the results are not really conclusive. The Owl seems to have during market jumps an edge. But otherwise the performance is quite similar. There are too less data available to answer this question with high evidence.

The Put versus Strangle question was in the first time-series clear. The Strangle performed better. This was not the case for the recent time-series. The strong rally created some problems at the upside. Driving a car by looking in the rear-mirror is a difficult task.
Further Work:
This working paper analyzed the effect of different hedging and exit approaches. An important open question is the identification of good entry points. The Kir does not enter the market if the IVTS is above 1.0. But otherwise the trader is on his own. One needs a reasonable measure for the risk-premium. At of this writing I have an extension of the filtered historic simulation proposed in [9] in my mind.

Note-1: In [2] I have declared this idea as forever death. But one should never say forever.
Note-2 in Rev. 1: There has been considerable progress in the meantime in this question. The results so far are at least not completely hopeless.

References: