Harvesting the Downside Beta Premium with the Implied Volatility Term Structure: 
The Cinderella Strategy. 
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Cinderella is a European folk tale embodying a myth-element of unjust oppression/triumphant reward. The title character is a woman living in unfortunate circumstances that are suddenly changed to remarkable fortune. Although both the story's title and the character's name change in different languages, in English-language folklore "Cinderella" is the archetypal name. The word “Cinderella” has, by analogy, come to mean one whose attributes were unrecognized, or one who unexpectedly achieves recognition or success after a period of obscurity and neglect. (http://en.wikipedia.org/wiki/Cinderella)

Abstract:
Ang, Chen and Xing have shown in [1] that stocks that covary strongly with the market during market declines have high average returns. The reward for bearing downside risk is not simply compensation for regular market beta, nor is it explained by coskewness or liquidity risk, or by size, value, and momentum characteristics. The downside beta premium is not for free. The drawdown is considerable amplified in times of market troubles. This working paper combines the results of Ang, Chen and Xing with a signal based approach developed in a previous working-paper. The implied-volatility-term-structure (IVTS) classifies market regimes. One invests only in the favorable regime fully into stocks with high downside beta. Due to this classification the downside beta premium is (almost) a free lunch.

Downside Beta:
One measure of downside risk is the downside beta (denoted by $b^-$):

$$b^- = \frac{\text{cov}(r_i, r_m | r_m < t_m)}{\text{var}(r_m | r_m < t_m)}$$  (1)

where $r_i$ ($r_m$) is security i’s (the market’s) return, and $t_m$ is a threshold. In the simplest case $t_m$ is Zero. It can also be set to the mean- or median return in the considered time window. The final effect on trading performance is minor. On simply calculates beta only on days when the market is down. Ang et al. calculated $b^-_i$ over the last year. One selects a portfolio of stocks with the highest downside beta. This portfolio is kept for one month. But in agreement with the results in [3] it is slightly better to omit the last month for the downside beta calculation.

Graphic-1 shows a simple application of this idea for the Nasdaq-100. The market is the ETF QQQ. Trading starts at 2010-01-01 till 2014-01-22 (the latest available data at this writing). There are 3 major adverse events in the considered time-range. The first is the flash crash at 2010-05-06. The next dangerous moment was the Japanese earthquake in March 2011. The third the crash in summer 2011.

One calculates on the last trading day of each month for all stocks the downside beta. The top-ten stocks are selected. In [1] the authors note, that stocks with a very high volatility have a lower downside beta premium. For this reason, only stocks with a volatility less than 1.9 times the QQQ are selected (the beta calculation sets the value for such stocks to zero. They are not considered in the final
Graphic-1: Nasdaq-100 Portfolio (orange), ETF-QQQ (yellow) from 2010-01-01 till 2014-01-22

Following [3] the volatility is calculated for the last 2 months with the Yang-Zhang-estimator [4]. The result is relative insensitive to the exact volatility scale factor. In a first attempt I used an absolute volatility limit of 30% (or 35%). But scaling the limit with the current market volatility seems to be more reasonable and gives also significantly better results.

The index starts with a value of 500,000$. The same value is used in previous working papers. One always invests the full index. According to [3] the weight of each stock was set to the inverse-volatility. This improves the performance slightly over an equal weighted portfolio. The difference is only minor, because all stocks have usually a relative high volatility near the upper volatility-bound.

The final win of the portfolio is 149.5%. The QQQ gains in the same time 98.5%. The drawdown beta premium is not for free. The max. relative drawdown during the summer 2011 crash is 24.5% (red-line in Graphic-1). The drawdown of the QQQ is only 16.1%.

Graphic-2 shows the same approach for the S&P-500. The market is in this case the SPY. The only difference is the volatility scaling factor. It is set to 3.0. I broad index as in relation a lower volatility than its constituents. The downside beta portfolio beats with 138.5% to 75.9% the SPY by a wide margin. But it losess in the summer 2011 crash 46.7%. The max. relative drawdown of the SPY is 18.6%. Trading the straight downside beta portfolio is only for the faint-hearted.
Graphic-2: S&P-500 Portfolio (orange), ETF-SPY (yellow) from 2010-01-01 till 2014-01-22

Graphic-3: Dow-Jones Portfolio (orange), ETF-DIA (yellow) from 2010-01-01 till 2014-01-22
Graphic-3 shows the Dow-Jones portfolio. For the Dow only the top-5 stocks are selected. The volatility scale factor is set to 1.75. The downside beta portfolio outperforms the DIA with 82.4% to 70.6%. The maximum relative drawdown is with 29.5% almost double as large as the drawdown of the DIA.

**The Implied Volatility Term Structure:**

The Implied Volatility Term Structure is generally defined as:

\[
\text{IVTS}(t) = \frac{\text{Short-Term-IV}(t)}{\text{Long-Term-IV}(t)} \quad (2)
\]

One selects on this curve points with readily available implied volatility measures. See [2] for a detailed study of the different IVTS measures. The best one was the VIX/VX30 ratio.

\[
\text{IVTS}(t) = \frac{\text{VIX}(t)}{\text{VX30}(t)} \quad (3)
\]

The VX30 is the price of a VIX future with a maturity of 30 calendar days. Usually such a future does not exist. In this case one calculates the weighted mean of the 1st and the 2nd nearest future. If the first future has a maturity of 20 days, and the second a maturity of 50 days, one calculates the VIX futures 30 value as \(2/3 \times \text{future}_1 + 1/3 \times \text{future}_2\).

According to previous studies ([2],[5]) it is beneficial to smooth the IVTS with a median-5 filter. The filter removes market overreactions. For a general statistical treatment of this type of filter see the nice monograph of G. Arce [6].

**The Cinderella Strategy:**

The Cinderella is a variant of the HeroRATs strategy developed in [2]. The HeroRATs strategy trades the VBK (Vanguard Small Cap Growth) and the TLT (iShares 20+ Year Treasury Bond) ETF. One fully invests into the VBK, if the IVTS defined in (3) is below 0.97. Between 0.97 and 1.10 one holds 50% of the VBK and 50% of the TLT. Above 1.10 one fully invests in the TLT. Graphic-4 shows the performance of the VBK.

*Note: The IVTS parameters were set to 3.0 and 10.0. One always invests fully in the VBK.*

The VBK gains in the 4 years 108.2%. The maximum relative drawdown is 28.9%. It happens in the summer 2011 crash. The overall picture is similar to the Graphics 1 to 3.

For the Cinderella the VBK is replaced by the Downside Beta Portfolio. Otherwise the rules are the same.

Graphic-5 shows the performance of the Cinderella Nasdaq-100 strategy. The plain Nasdaq-100 portfolio had the maximum relative drawdown on 2011-08-16 (red line in Graphics-1 and 5). The Cinderella is already wining at this time. It switches 2 weeks before to a full investment in TLT. Cinderella is loosing somewhat in the foreshocks of the crash. The maximum relative drawdown is with 18.2% at 2010-08-31. This is the end of the aftershocks of the flash-crash. In August 2010 nothing special happened. The IVTS was low, the market went sideways down. The IVTS smells only serious danger.

*Note: The real HeroRATs are trained to detect land mines.*

The final win of the Cinderella Nasdaq-100 are impressive 177.8%.
Graphic-4: ETF VBK (orange) and SPY (yellow) from 2010-01-01 till 2014-01-22

Graphic-5: Cinderella Nasdaq-100 (orange), ETF-QQQ (yellow) from 2010-01-01 till 2014-01-22
Graphic-6 shows the performance of the Cinderella S&P-500. The max. relative drawdown is at 2012-07-04 with 24.1% (red line in Graphic-6). Nothing special happens at this time. The market just went sideways down and the large downside betas accelerated the decline. But the final win is with 200.4% also impressive.

The Dow-Jones Cinderella in Graphic-7 combines a relative modest drawdown of 10.6% with an overall performance of 118.2%.

Graphic-8 shows for comparison reasons the performance of the HeroRATs strategy. The picture is similar. The maximum relative drawdown is 13.4% at 2010-08-24. The HeroRATs suffers also somewhat in the aftershocks of the flash-crash. The overall performance is 148.1%.
Graphic-7: Cinderella Dow-Jones (orange), ETF-DIA (yellow) from 2010-01-01 till 2014-01-22

Graphic-8: HerRATs Strategy (orange), ETF-SPY (yellow) from 2010-01-01 till 2014-01-22
Conclusion:
This work confirms the results of Ang, Chen and Xing from 2005. There is a large downside beta premium. But the premium is not for free. This premium can be harvested at medium risk with the Cinderella. The direct alternative is the HeroRATs strategy. The Cinderella and the HeroRATs are similar approaches. The HeroRATs is easier to implement. The details of portfolio construction are solved by Vanguard. The HeroRATs has also a lower drawdown than the Cinderella Nasdaq or the Cinderella S&P-500. On the other hand both Cinderellas outperform the HeroRATs considerable. The fairy prince suffers from the paradox of choice ([7]). There are 3 attractive brides. But it is difficult to say which is the best one.

References: